Comment on "Position-dependent effective mass Dirac equations with PT- symmetric and non - PT- symmetric potentials" [J. Phys. A: Math. Gen. 39 (2006) 11877–11887]

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Abstract

Jia and Dutra (J. Phys. A: Math. Gen. 39 (2006) 11877) have considered the one-dimensional non-Hermitian complexified potentials with real spectra in the context of position-dependent mass in Dirac equation. In their second example, a smooth step shape mass distribution is considered and a non-Hermitian non - PT- symmetric Lorentz vector potential is obtained. They have mapped this problem into an exactly solvable Rosen-Morse Schrödinger model and claimed that the energy spectrum is real. The energy spectrum they have reported is pure imaginary or at best forms an empty set. Their claim on the reality of the energy spectrum is fragile, therefore.

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Very recently, Jia and Dutra [1] have presented a new procedure to construct a set of non-Hermitian potentials with real spectra for the one-dimensional Dirac particle endowed with position-dependent mass. They have reported that a Lorentz vector potential of the form

$$V(x)=\frac{i}{2}\frac{1}{M(x)}\frac{dM(x)}{dx} \tag{1}$$

would imply a Schrödinger-like Dirac equation

$$-\frac{d^2}{dx^2}\varphi(x)+V_{eff}(x)\varphi(x)=E^2\varphi(x), \tag{2}$$

where $V_{eff}(x) = M(x)^2$, $M(x)$ and E are the mass and the real energy of the Dirac's particle, respectively, and $\varphi(x)$ is the upper component of the Dirac spinor (eqs. (1)-(9) in [1]).

In their second example (section 3.2 in [1]) they have considered a smooth step shape mass distribution

$$M(x) = M_\circ (1 + \eta \tanh \alpha x) \tag{3}$$

that led to a non-$\mathcal{PT}$-symmetric non-Hermitian Lorentz vector potential

$$V(x) = \frac{i}{2} \frac{\alpha\eta \operatorname{sech}^2 \alpha x}{(1 + \eta \tanh \alpha x)} \tag{4}$$

where η is a small parameter that satisfies the condition $|\eta| \leq 1$ to get a positive mass distribution. Under such settings, Eq.(2) reads a Rosen- Morse-type Schrödinger problem [2]. Employing the SUSQM method [3], Jia and Dutra [1] have reported that the reality of energy spectrum

$$E_n = \pm \sqrt{M_\circ^2 (1 + \eta^2) - \frac{\eta^2 M_\circ^4}{\alpha^2 (n + \delta_1)^2} - \alpha^2 (n + \delta_1)^2}, \quad n = 0, 1, 2, \cdots, \tag{5}$$

with

$$\delta_1 = \frac{1}{2} \left(1 - \sqrt{1 + \frac{4\eta^2 M_\circ^2}{\alpha^2}} \right), \tag{6}$$

is guaranteed if and only if

$$M_\circ^2 (1 + \eta^2) \geq \frac{\eta^2 M_\circ^4}{\alpha^2 (n + \delta_1)^2} + \alpha^2 (n + \delta_1)^2. \tag{7}$$

In this comment we shall show that even with the lowest possible value of the principle quantum number n (i.e., $n = 0$) the inequality in (7) can not be satisfied. Hence, the reality of the spectrum in (5) is fragile (of course within the parameters' settings reported in [1] for this particular case).

To do so, we start with the introduction of a positive parameter $\Lambda = \alpha / M_\circ \geq 0$ to recast, with $n = 0$, the inequality (7) as

$$f(\eta, \Lambda) = (1 + \eta^2) - \frac{4\eta^2}{\left(\Lambda - \sqrt{\Lambda^2 + 4\eta^2} \right)^2} - \frac{1}{4} \left(\Lambda - \sqrt{\Lambda^2 + 4\eta^2} \right)^2 \geq 0. \tag{8}$$

Then we shall now test the validity of $\mathbb{R} \ni f(\eta, \Lambda) \geq 0$ for $-1 \leq \eta \leq 1$ and $\Lambda \geq 0$, where $\eta, \Lambda \in \mathbb{R}$.

A 3D plot, in Figure 1, of $f(\eta, \Lambda)$ with $\Lambda \in [0, 10]$ clearly indicates that $0 \geq \mathbb{R} \ni f(\eta, \Lambda) \ngeq 0$ over the physically acceptable parametric values. Therefore, the energy spectrum in (5) can only be pure imaginary (i.e., $E_n \in \mathbb{C}$ and $E_n \notin \mathbb{R}$)

Figure 1: A 3D plot of $f(\eta, \Lambda)$ with $\Lambda \in [0, 10]$ and $-1 \leq \eta \leq 1$.

References

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